CALCULATING THE EMISSIVITY OF SEMITRANSLUCENT MATERIALS

I. V. Men' and O. A. Sergeev

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A procedure is outlined for calculating the emissivity (spectral and integral, directional and hemispherical) of a semitranslucent plane layer on an opaque base, taking into account temperature gradients in the layer as well as polarization of the rays and multiple reflections at the boundary. The values of emissivity are given for grades K8 and TF1 glass.

As is well known, radiation in a semitranslucent medium is spatial in character: not only the surface of the material radiates but also the inner layers. For this reason, the emissivity of semitranslucent materials depends on the thickness of the radiating specimen and on the temperature gradients inside it. Under these conditions, any direct emissivity measurement by conventional methods becomes meaningless for this class of materials, since a value obtained in test may apply to the given specimen but not as an unbiased property of the material. It is more appropriate here to measure the absorptivity of the material, from which the emissivity of any semitranslucent system can then be calculated. General problems relating to this approach have been discussed earlier in [1], and here we will analyze the actual implementation of the mathematical procedure.

The emissivity of semitranslucent materials has been determined for several specific cases in [2-6] on the basis of absorptivity measurements. Isothermal plates were studied in [2, 5], but only the spectral emissivity was considered in [2] and only the integral emissivity in the direction normal to the surface was considered in [5]. The normal emissivity of a layer with a linear or parabolic temperature distribution was calculated in [3, 4], but without taking into account the polarization and the multiple reflections of rays. In [6] the polarization of light was also disregarded, but the temperature drop was taken into consideration in determining the integral directional and the integral hemispherical emissivity of a semitranslucent dielectric layer deposited on a metallic substrate. The author of [6] used the "gray" approximation, which does not account for the optical spectrum characteristics of the material.

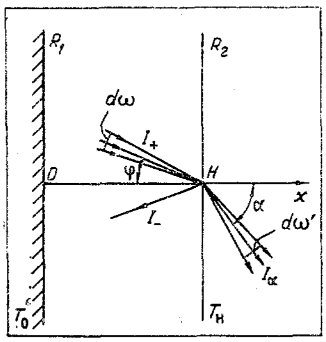

Fig. 1

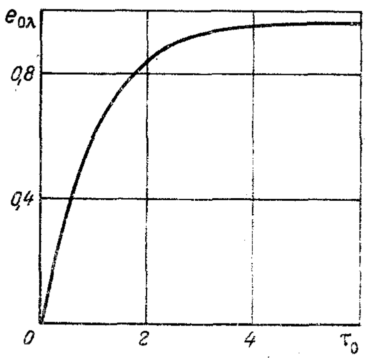

Fig. 2

Fig. 1. Pertaining to the derivation of basic formulas.

Fig. 2. Spectral normal emissivity of grade K8 glass, as a function of the optical layer thickness ($\lambda = 2.7\ \mu$m). Optical thickness $\tau_0 = kH$.

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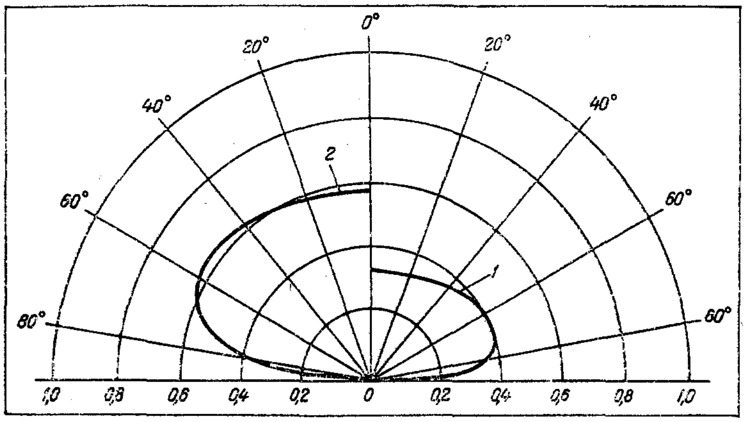

Fig. 3. Angular distribution of spectral emissivity for grade TF1 glass (λ = 2.7 μm, H = 1 cm): t = 20°C (1), 550°C (2).

We will consider a radiating medium represented by a homogeneous semitranslucent plate of thickness H (Fig. 1) whose optical spectrum characteristics (n_λ and k_λ) have been measured. For the sake of generality, we will assume in this analysis that the lower surface of the plate lies on an opaque body (substrate) whose temperature T_s and reflective properties are known. The temperature field T(x) in the layer may be of any arbitrary character. It has been shown in [1, 6] that T(x) under such conditions is found by a simultaneous solution of the radiant-energy transfer equation and of the radiative–convective heat transfer equation. In our case T(x) will be assumed already known.

Since there are no radiation sources on the surface, hence (Fig. 1)

$$I_\alpha(\lambda)\cos\alpha d\omega' = I_+(H, \varphi, \lambda)\cos\varphi d\omega P(\lambda), \tag{1}$$

where P(λ) is the transmittivity of the surface, $I_+(H, \varphi, \lambda)$ is the intensity of radiation impinging on the upper plate surface from below at an angle φ, $I_\alpha(\lambda)$ is the intensity of radiation leaving the layer at an angle α, and both these angles are related according to the law $\sin\alpha = n_\lambda \sin\varphi$. The further analysis will apply to a definite wavelength. For clarity, the symbol λ will be omitted from subscripts. According to the refraction law, we find from (1);

$$I_\alpha = \frac{P}{n^2} I_+(H, \varphi). \tag{2}$$

With polarization of light taken into account, relation (2) simplifies to

$$I_\alpha = I_{+s}(H, \varphi)\frac{P_s}{n_s^2} + I_{+p}(H, \varphi)\frac{P_p}{n_p^2}, \tag{3}$$

where the subscripts refer to the s- and p-component of polarized light respectively. In order to find $I_{+s}(H, \varphi)$ and $I_{+p}(H, \varphi)$, we solve the light transmission equation. If the medium is isotropic, then $k_s = k_p = k$ and $n_s = n_p = n$, and the radiation coefficient for a given material locally under thermal equilibrium without extraneous radiation sources will be determined from the equality $j_s(T) = j_p(T) = kn^2 I_B(T)/2$, where $I_B(T)$ denotes the spectral Planck function for temperature T. In such a case the solution to the transmission equation are analogous for both components s and p. We will examine one of them (omitting the subscript s, p). The equation of light transmission

$$\frac{dI}{dx} = -\frac{kI}{\cos\varphi} + \frac{j(T)}{\cos\varphi} \tag{4}$$

will be solved, as in [6], separately for angles $\varphi < \pi/2$ and $\varphi > \pi/2$: the former corresponding to light which travels from below upward (denoted by the "+" sign) and the latter corresponding to light which travels from above downward (denoted by the "−" sign). Considering the presence of a substrate, we write the boundary conditions as

$$\begin{aligned} I_+(0, \varphi) &= B(\varphi) + R_1(\varphi) I_-(0, \varphi), \\ I_-(H, \varphi) &= R_2(\varphi) I_+(H, \varphi); \end{aligned} \tag{5}$$

$B(\varphi)$ is the luminance of the substrate in a medium with a refractive index n; $B_s = B_p = \varepsilon(\varphi) I_B(T_s) n^2/2$. The solution to Eq. (4) with (5) is

$$I_+(H, \varphi) = \int_0^H \frac{j(\xi)}{\cos\varphi} e^{-\frac{k(H-\xi)}{\cos\varphi}} d\xi + \left[1 - R_1 R_2 e^{-\frac{2kH}{\cos\varphi}}\right]^{-1} \left[B_1 e^{-\frac{kH}{\cos\varphi}} + R_1 \int_0^H \frac{j(\xi)}{\cos\varphi} e^{-\frac{k(H+\xi)}{\cos\varphi}} d\xi + R_1 R_2 \int_0^H \frac{j(\xi)}{\cos\varphi} e^{-\frac{k(3H-\xi)}{\cos\varphi}} d\xi\right]. \quad (6)$$

Reflectivities R_1 and R_2 are calculated according to the Fresnel formula for the respective component, with absorption in the layer taken into account [8] (light impinges from inside on the media boundary). Inserting these values into (2) and also considering that $R_s = 1-R_{2s}$, $R_p = 1-R_{2p}$, we obtain after a few transformations:

$$I_\alpha = \varepsilon(\varphi) I_B(T_s) e^{-\frac{kH}{\cos\varphi}} \left[\frac{M_s}{2}(1-R_{2s}) + \frac{M_p}{2}(1-R_{2p})\right] + \frac{k}{\cos\varphi} \int_{\xi=0}^H I_B[T(\xi)] \left\{\frac{1}{2}(1-R_{2s}) e^{-\frac{k(H-\xi)}{\cos\varphi}} + \frac{M_s}{2}(1-R_{2s}) \left[R_{1s} e^{-\frac{k(H+\xi)}{\cos\varphi}} + R_{1s} R_{2s} e^{-\frac{k(3H-\xi)}{\cos\varphi}}\right] + \frac{1}{2}(1-R_{2p}) e^{-\frac{k(H-\xi)}{\cos\varphi}} + \frac{M_p}{2}(1-R_{2p}) \left[R_{1p} e^{-\frac{k(H+\xi)}{\cos\varphi}} + R_{1p} R_{2p} e^{-\frac{k(3H-\xi)}{\cos\varphi}}\right]\right\} d\xi. \quad (7)$$

Here

$$M_s = \left[1 - R_{1s} R_{2s} e^{-\frac{2kH}{\cos\varphi}}\right]^{-1}; \quad M_p = \left[1 - R_{1p} R_{2p} e^{-\frac{2kH}{\cos\varphi}}\right]^{-1}.$$

In this way, the spectral directional emissivity $e_{\alpha,\lambda}$ of the layer – with all the said factors taken into account – is determined from the equality

$$e_{\alpha,\lambda} = I_\alpha / I_B(\lambda, T_L), \quad (8)$$

where I_α is found from (7) for any given wavelength λ and direction α, while angles α and φ are related according to the refraction law.

It is easy to show that, knowing how to calculate $e_{\alpha,\lambda}$, one can find the spectral hemispherical emissivity according to the formula

$$e_{hs,\lambda} = 2 \int_{\alpha=0}^{\pi/2} e_{\alpha,\lambda} \sin\alpha \cos\alpha \, d\alpha, \quad (9)$$

and integration over all wavelengths will yield the integral directional emissivity:

$$e_{\alpha,i} = \frac{\pi \int_{\alpha=0}^{\infty} e_{\alpha,\lambda} I_B(\lambda, T_L) \, d\lambda}{\sigma T_L^4}. \quad (10)$$

Finally, with the aid of (10), we find the integral hemispherical emissivity

$$e_{hs,i} = 2 \int_{\alpha=0}^{\pi/2} e_{\alpha,i} \sin\alpha \cos\alpha \, d\alpha. \quad (11)$$

The special cases considered in [2-6] can be treated in terms of the results which we have obtained here. Thus, for instance, for an isothermal layer without substrate we let $T(\xi) \equiv T$, $\varepsilon(\varphi) = 0$, $R_{1s} = R_{2s} = R_s$, and $R_{1p} = R_{2p} = R_p$ in expression (7). Furthermore, $j(T)$ is taken outside the integral sign. Performing the integration then, we obtain

$$I_\alpha = \frac{1}{2} I_B(T) \left(1 - e^{-\frac{k_\lambda H}{\cos\varphi}}\right) \left(\frac{1-R_s}{1-R_s e^{-\frac{k_\lambda H}{\cos\varphi}}} + \frac{1-R_p}{1-R_p e^{-\frac{k_\lambda H}{\cos\varphi}}}\right), \quad (12)$$

from where, after dividing by $I_B(T)$, we find for $e_{\alpha, \lambda}$ the expression which R. Gardon has obtained in [2].

The emissivity of various semitranslucent materials was calculated according to the procedure shown here. The spectral normal emissivity (for $\lambda = 2.7$ μm) is shown in Fig. 2 for grade K8 glass, as a function of the optical layer thickness. The emissivity as a function of the observation angle is shown in Fig. 3 for a 1 cm thick plate of grade TF1 glass. At that particular wavelength, evidently, there is a strong temperature-dependence here. Deviations from the Lambert Law are appreciable. They become smaller at elevated temperatures, however, because the absorptivity and thus also the optical thickness of the plate increase with temperature.

NOTATION

$e_{\alpha, \lambda}$	is the spectral directional emissivity;
$e_{hs, \lambda}$	is the spectral hemispherical emissivity;
$e_{\alpha, i}$	is the integral directional emissivity;
$e_{hs, i}$	is the integral hemispherical emissivity;
n	is the refractive index;
k	is the absorptivity;
$j(T)$	is the radiation coefficient for the medium (spectral volume intensity of radiation);
R	is the reflectivity;
$I(\lambda)$	is the radiation intensity;
$I_B(\lambda, T)$	is the Planck function;
H	is the layer thickness
$\tau_0 = kH$	is the optical layer thickness;
ρ	is the transmittivity of the media boundary;
$B(\varphi)$	is the luminance of substrate along angle;
ε	is the emissivity of substrate;
T_s	is the temperature of substrate;
T_0, T_L	is the temperature of the layer boundaries;
σ	is the Stefan constant;

Subscripts:

1, 2	denote the lower and upper boundary;
s, p	denote the components of polarized light;
+, −	denote the light traveling in two opposite directions spectral values (wavelength);
α, φ	denote the angular functions corresponding to angles α, φ from the normal to a surface.

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